

THE PHYSICS OF COMETARY NUCLEI

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## NOTE ON THE STRUCTURE OF COMET NUCLEI

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### ABSTRACT

The recent developments in cometary studies suggest rather low mean densities and weak structures for the nuclei. They appear to be accumulations of fairly discrete units loosely bound together, as deduced from the observations of Comet Shoemaker-Levy 9 during its encounter with Jupiter. The compressive strengths deduced from comet splitting by Öpik and Sekanina are extremely low. These values are confirmed by theory developed here, assuming that Comet P/Holmes had a companion that collided with it in 1892. There follows a short discussion that suggests that the mean densities of comets should increase with comet dimensions. The place of origin of short-period comets may relate to these properties.

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Comet Densities, Impacts, Compressive Strengths, Sources and General Structure.

## Introduction

The understanding of the physical structure of comet nuclei has been slow in developing. The surprising variations in the concepts of comets since Newton's day are well documented by Sekanina (1996a) and need not be repeated here. Estimates of the nuclear density are relatively recent, rising from extremely low values to  $1.5 \text{ gm/cm}^3$  (Whipple 1950) based on the icy conglomerate model. With a solar mix of heavy elements, Öpik (1966) adopted  $\rho = 2 \text{ gm/cm}^3$  in calculating the crushing strength,  $s_c$ , for several sun-grazing comets in the range  $2.0 \times 10^4$  to  $1.9 \times 10^6 \text{ dyne/cm}^2$  on the basis of tidal forces. In that paper Öpik introduced the description of comets as "heaps of rubble" and found their crushing strengths greater than those of "dust balls" ( $s_c \sim 10^4 \text{ dyne/cm}^2$ ).

Because of fairly frequent comet splitting, the concept of comets as heaps of rubble has become fairly popular with astronomers and has lead to estimates of density as low as  $0.2 \text{ gm/cm}^3$ . The Giotto and Luna space missions to Halley's comet indicated a geometric albedo of about 0.04, a bit darker than the moon, and showed that Halley's comet was elongated, somewhat of a peanut shape. Unfortunately, the spacecraft did not approach the comet closely enough to measure its gravitational attraction and, therefore, mass and density.

## Recent Progress

The splitting of Comet Shoemaker-Levy, 9 and the influx of its 21 pieces and debris into Jupiter's atmosphere for a week during July 1994 combined to produce considerably more

information about comet structure. From their theories of tidal disruption of the comet by Jupiter, based on the orbit by Yeomans and Chodas (1993), Ashphaug and Benz (1994), and Solem (1994), deduced that the original comet has a density of  $0.5 \text{ gm/cm}^3$  with a diameter estimated at 1.5 and 1.8 km respectively. Because the literature on the Jupiter collision is so extensive, I shall limit further relevant references mostly to the symposium book edited by Noll, Weaver, and Feldman (1996) and the article in it by Sekanina (1996b) on the tidal breakup.

Ashphaug and Benz and, also, Solem assumed that the comet originally had no internal cohesive strength and was held together solely by gravity. Their solution for an original body of only 1.5 to 1.8 km in diameter is strongly refuted by observations of the brightnesses of the individual pieces by the Hubble Space Telescope (HST). Weaver et al. (1993) assumed a geometric albedo of 0.04 and from the HST observations of magnitudes deduced that the diameter of 11 pieces ranged from 2.5 to 4.3 km. The combined spherical volume of these pieces adds up to a diameter of  $\sim 7.7$  km and, of course, does not include the added volume of ablated ices, dust and smaller pieces. Thus the original diameter must have been  $\sim 10$  km.

Sekanina concludes that the comet must have consisted of many individual bodies, a number of them with some internal cohesive strength. This is attested to by the fact that the brighter (larger) fragments were not elongated by Jupiter's tidal forces before they entered the denser atmosphere. Weissman (1994) had pointed out that strengthless fragments would spread out and not make great explosions in Jupiter's atmosphere. Sekanina (1996)

believes that the original tidal breakup could be better described physically by the relation by Aggarwal and Oberbeck (1974) and Dobrovolskis (1990):

$$\frac{\rho}{\rho_o} \left( \frac{\Delta}{R_o} \right)^3 < 2, \quad (1)$$

where  $\rho$  is the density of the spherical body at a distance  $\Delta$  from the planet of density  $\rho_o$  and radius  $R_o$ . This relation is derived by the tidal separation of the two imagined hemispheres. The central pressure and radius of the spherical body do not enter the equation. Equation (1) leads to  $\rho < 1.1 \text{ gm/cm}^3$  for the progenitor of Comet Shoemaker-Levy 9 and  $< 0.34 \text{ gm/cm}^3$  for 16 P/Brooks 2. In a sense this limit might be considered also as a sort of upper limit because it assumes sphericity for the tidally broken body, non-rotating. A low density highly elongated body in rotation could well be more easily torn apart.

These arguments point rather uncertainly back to mean densities of an order of  $0.5 \text{ gm/cm}^3$ . Weissman (1995), for example, adopts the value  $0.6 \text{ gm/cm}^3$ . Sekanina prefers a somewhat lower value,  $\sim 0.2 \text{ gm/cm}^3$ , while Rickman (1986) settled on  $\rho = 0.28 \text{ gm/cm}^3$ , a value used in calculation by Greenberg et al. (1995) in determining the tensile strength of cometary nuclei.

## **The Crushing Strength of the Companion of Comet 17 P/Holmes**

In November 1892 Comet P/Holmes exhibited a massive flare that led to its discovery. A

month later it had faded 7–8 magnitudes, and then, in January 1893, it flared again about six magnitudes. I have presented (Whipple, 1984) some evidence that the first outburst could have been produced by a companion satellite that brushed the major nucleus to produce the first flare. Then 73 days later it collided with the nucleus to produce the second . The evidence suggested a rotation period of  $16^h.3$  for the nucleus, retrograde with respect to the orbit, and also that the second encounter both created an active area and also reactivated the area of the first encounter.

Since its two great outbursts, the comet, having a period of  $6^y.9$  to  $7^y.4$  and orbit inclination of  $19^\circ.2$  to  $20^\circ.8$ , has behaved like an average short-period comet. The magnitude of its nucleus measured by E. Roemer at 3 solar distances of 2.4 to 3.2 AU, range from 15.0 to 15.7 with a mean of 15.3 at 1 AU from Sun and Earth. This value for 1 P/Halley is about 15.0 making the current spherical radius of 17 P/Holmes about 4.4 km if that of 1 P/Halley is taken as 5.0 km effectively.

The resulting encounter velocity of the companion is about equal to the velocity of escape from the assumed spherical shape (i.e.  $\sim 2.5\text{m/sec}$ ) (Table 1), slightly smaller because of its possible shape and the eccentricity of its orbit and slightly larger because of a rotating nucleus.

To derive a crude theory, suppose a cube of side  $x$  flows into a very much larger similar comet body at velocity  $v_0$ . Suppose the cube stops after crushing  $1/n$  of its length, i.e. after penetrating a distance  $x/n$  both into the cube, and into the main comet body. The collision

lasts for an interval of time,  $t_e$ , until the back of the cube stops. Thus

$$t_e = 2x/nv_o. \quad (2)$$

The momentum,  $\rho x^3 v_o$ , is applied in time  $t_e$  so that the crushing strength,  $s_c$ , equal to the force/area, becomes momentum/time/area or

$$s_c = (\rho x^3 v_o) \left( \frac{nv_o}{2x} \right) \left( \frac{1}{x^2} \right) = \frac{n\rho v_o^2}{2}. \quad (3)$$

This theory is obviously too simplistic for at least three reasons: 1) the negative acceleration increases with time; 2) crushing causes the material to spread out near the main body; and 3) the lagging portion of the satellite is not crushed.

The variation in the negative acceleration can be approximated by assuming that it varies linearly with time ( $\sim at$ ). In this case

$$-v_o = -\int_0^t (at) dt_x = -\frac{1}{2}at^2, \quad (4)$$

so that

$$v = v_o - \frac{1}{2}at^2. \quad (5)$$



The distance,  $D = 2x/n$ , becomes

$$D = \int_0^{t_e} v dt = \int_0^{t_e} (v_o - \frac{1}{2}at^2 dt), \quad (6)$$

or

$$\frac{2x}{n} = v_o t_e - \frac{1}{6}at_e^3. \quad (7)$$

But, from Eq. 4,

$$a = \frac{2v_o}{t_e^2}. \quad (8)$$

Hence

$$\frac{2x}{n} = t_e(1 - \frac{1}{3})v_o \quad (9)$$

or

$$t_e = \frac{3x}{nv_o}, \quad (10)$$

so that

$$s_c = \frac{n\rho v_o^2}{3}. \quad (11)$$

The spreading effect can be included by an arbitrary spreading factor of  $(1 - 1/n)$ , leading finally to

$$s_c = \left(1 - \frac{1}{n}\right) \frac{n\rho v_o^2}{3}. \quad (12)$$

Because tiny soft particles will not crush when impacting at low velocities, one may be surprised that the mass of the comet satellite does not enter Eq. 12. In fact, it does enter implicitly, in the sense that Eq. 12 becomes valid only when the mass is great enough to induce a great amount of crushing. Because the actual mass of the satellite must amount to at least a number of tons to produce the observed activity of 17 P/Holmes, the requirement is well satisfied.

Table I lists values of  $s_c$  from Eq. 12 for various densities and assumed values of  $n$ , the fraction of the volume that is reduced by crushing ( $v_o = 2.5$  m/sec).

Table I  
Compressive Strength

$\rho$ (gm/cm <sup>3</sup> )	n	$s_c$ dyn/cm <sup>2</sup>
0.2	2	$4.2 \times 10^3$
0.28	3	$1.2 \times 10^4$
0.5	5	$4.2 \times 10^4$
1.0	10	$1.9 \times 10^5$
1.5	20	$5.9 \times 10^5$

The Table I values of  $s_c$  are comparable to those calculated by Öpik, which range from  $2.0 \times 10^4$  to  $1.9 \times 10^6$  dyn/cm<sup>2</sup>, and the tensile strengths,  $T$ , calculated by Sekanina (1993) for pieces of comet Shoemaker-Levy 9, from his assumption that  $T/\rho = 15000 - 20000$  dyn cm/g. Using the mean value  $T/\rho = 17,500$  dyn cm/g, the range of  $T$  corresponding to the densities in Table I becomes  $3.5 \times 10^3$  to  $2.6 \times 10^4$  dyn/cm<sup>2</sup> for Sekanina's solution of tensile strength.

In their theoretical calculation of the tensile strength of cometary nuclei, Greenberg et al. (1995) derive the value  $T = 2.7 \times 10^3$  dyn/cm<sup>2</sup> with their assumed value of  $\rho = 0.28$  g/cm<sup>3</sup>, somewhat less than the compressive strength of Table I.

### The Mean Densities of Cometary Nuclei and General Conclusions

Following the general idea that comets first form by the accumulation of interstellar dust and then by impacts, the mean density of the individual comet will depend primarily on its total mass or size. The major physical factors involved are 1) gravitational compression, and 2) the velocities of impacts during accumulation. Probably lesser roles are played by induced

rotation from impacts and irregular shapes of the resulting nuclei by impacts. Possibly Pluto represents the maximum density if  $\rho = 1.8$  to  $2.1 \text{ gm/cm}^3$ . For very large comets, heating by radioactivity will force more volatile materials out from the center (Whipple and Stefanik, 1966) but will not much affect the comet's mean density until it has aged by ablation. The nucleus of a very large comet in its later stages may reach the maximum value of density.

By comparing Tables I and II, we see that central compression might increase the mean density of a comet of nominal density  $0.2 \text{ gm/cm}^3$  at a radius nearing 10 km, while for larger nominal densities the effect of compression begins at  $R > 10 \text{ km}$ .

On the other hand, if compression during impact begins at impact velocities of near 2 to 2.5 m/sec, rather small comets of radii about 5 km, would have their outer layers compressed at mean density  $< 0.3 \text{ gm/cm}^3$ , and for somewhat smaller radii at higher mean densities.

Thus it seems that most of the observable comets may have rather low mean densities such as deduced for Comet Shoemaker-Levy 9 and low tensile and compressive strengths as discussed earlier. All of this rests on the assumption that impact velocities are not much greater than the velocities of escape.

A contrary point of view stems from the calculations and deductions of Farinella and Davis (1996), who conclude that short-period comets are derived as broken pieces from encounters among very much larger comets ( $R \sim 200 \text{ km}$ ) in the Kuiper belt. These secondary comets would have surely been compressed in the breakups at impact velocities of the order of 100 m/sec, and many would have been composed of pieces already compressed by internal

pressures.

If, indeed, as Holman and Wisdom conclude, the short-period comets do come primarily from the Kuiper belt, it would seem that they are fairly pristine and not pieces of huge comets.

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TABLE II

Velocity of Escape and Central Pressure\*

$\rho(\text{gm/cm}^3)$	0.2		0.28		0.5		1.0	
R(km)	$V_\infty$	$P_c$	$V_\infty$	$P_c$	$V_\infty$	$P_c$	$V_\infty$	$P_c$
1.0	0.3	$5.6 \times 10$	0.4	$1.1 \times 10^2$	0.5	$3.5 \times 10^2$	0.7	$1.4 \times 10^3$
2.0	0.7	$2.2 \times 10^2$	0.8	$4.4 \times 10^2$	1.1	$1.4 \times 10^3$	1.5	$5.6 \times 10^3$
5.0	1.7	$1.4 \times 10^3$	2.0	$2.7 \times 10^3$	2.6	$8.7 \times 10^3$	3.7	$3.5 \times 10^4$
10.0	3.3	$5.6 \times 10^3$	4.0	$1.1 \times 10^4$	5.3	$3.5 \times 10^4$	7.5	$1.4 \times 10^5$
50.0	16.7	$1.4 \times 10^5$	19.8	$2.7 \times 10^5$	26.4	$8.7 \times 10^5$	37.4	$1.4 \times 10^6$
100.0	33.4	$5.6 \times 10^5$	39.6	$1.1 \times 10^6$	52.9	$3.5 \times 10^6$	74.8	$1.4 \times 10^7$

\* For spheres where  $V_\infty(\text{m/sec})$  is the velocity of escape and  $P_c(\text{dyn/cm}^2)$  is the central pressure.